

Optimization Based on 3D CFD Simulations of An Axial Turbine from a Small Gas Turbine Engine

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Abstract

The performance of gas turbines has had a great improvement over the last decades. Modern turbo machinery are present in many industries for several applications as the automotive industry, oil and gas and aviation. Aeronautical turbines are used to generate enough thrust to make actual airships take flight using a big variety of technologies and are responsible for more than 5% of all world's liquid fuel consumption [1]. The state of the art for this application demands the most powerful present resources in order to achieve small increases of performance that are more than relevant to the coasts associated to the need to take people and products across big distances in small time. In this scenario, CFD reveals itself as a primary tool for the betterment of current equipment aiming efficiency, power and fuel consumption improvements.

This study aims to optimize an axial turbine used in a small gas turbine engine with 5 kN of thrust developed by the Aeronautical Technological Institute (ITA). Commercial numerical simulation software are used in combination to evolutionary genetic algorithm NSGAII to handle multiobjective design. With the geometry file obtained from a previous work, mesh generation and all subsequent steps were performed computationally using ANSYS Design Modeler, TurboGrid, CFX and CFD POST tools, in parallel to the use of the ANSYS Design Explorer and ESTECO's ModeFrontier, a multidisciplinary and multi task tool. The fluid is assumed incompressible and the turbulent flow in the steady-state regime.

Keywords: Turbo-machinery, Fuel-consumption-reduction, Optimization, CFD, ANSYS, Multi-objective, ModeFrontier



INTRODUCTION

In the present work, the axial turbine used to supply power to the compressor in the propulsion system of an aircraft, known as gas turbine engine, was studied and a methodology was created for the optimization of this turbo machine in its early design phase. A suitable design of an axial turbine is aimed at generating power, efficiency and a good compromise with its performance. One of the great challenges is undoubtedly the adequacy and good dimensioning of its axial channel, from the input to the output of the turbo machine. Because it is a time-consuming and complex task to change and analyze the flow at each geometric change in the turbine, the use of support tools is undoubtedly very interesting to the designer. One of these resources is the use of optimization algorithms coupled with the design tools.

A methodology has been developed to use these tools in a coupled way with the aim of providing interesting results to the designer, who can analyze different solutions for a given problem. The choice of the most appropriate design, at the end of the optimization process, is strongly dependent on what is desired of the turbo machine. The optimization technique performs searches through objective functions and constraints, adopted by the user, with the use of specific algorithms. Thus, the computational tools of the compressible and turbulent flow through the turbine were coupled with optimization models. The flow was calculated using Computational Fluid Dynamics (CFD) techniques. With the solutions of the characteristics of the flow, it is possible to determine the characteristics of operation of the turbine.

AXIAL TURBINE FUNDAMENTAL CONCEPTS

Turbines are components that transform thermal and pressure energy into mechanical energy through the kinetic energy of the flow through the grid of the rotating system. The generated mechanical energy can be used in several ways, such as triggering another device (eg a compressor or a pump) or to move a generator.

Essentially, the two most common types of turbines are: the radial and the axial. There are mechanisms to determine which type is most suitable for a particular application, for example using the speed or specific rotation diagram used to choose the type of machine. In general, for low flow values, radial turbines are more indicated.

As a basic operating principle, the hot gas from the combustion chamber passing through the axial turbine is subjected to varying the amount of movement, raising the kinetic energy of the relative flow and as a consequence generating movement (rotation) of the rotor. This rotor grille then acquires mechanical power through the shaft that is absorbed by the compressor.



COMPUTATIONAL FLUID DYNAMICS – CFD

Computational Fluid Dynamics is the area of engineering that determines the general equations of fluid mechanics (continuity, momentum and energy) through approximate numerical solutions. The initial development of this technique dates back to the late 1960s, when Brian Spalding worked with his students to transform CFD from a mathematical science to a rich resource for engineers to solve real problems [1]. From established theoretical concepts, numerical and computational methods are used to calculate complex problems that would be difficult to solve by means of analytical methods. This technique uses the theory of transport, heat transfer, chemical reactions and other fluid dynamics phenomena to assist the industry in the research and development of projects. CFD provides detailed and accurate flow and fluid behavior information, such as pressure, velocity and temperature distributions, forces and torques, phase distributions, flow paths, recirculation flow zones, among others.

Its application takes place in a wide range of segments, such as aerospace, automotive, biomedical, energy, chemical processes, electronic components, and can be used in all stages of the engineering process, from concept and design of new products to phase optimization, fault handling and re-design.

GOVERNING EQUATIONS

In mathematical terms we can introduce the governing equations used by the CFD technology by means of a control volume.



Figure 1: Space defined system [2]

Through the balance in this control system and the use of the divergence theorem it is possible to arrive at the differential form of the transport equation.

$$0 = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{v} \tag{(1)}$$



The equation can be presented in terms of mass, momentum and energy after the imbalance on the control system. After some rearrangement it is possible to express the general transport equation or general transport equation of \emptyset variable:

$$\frac{\partial}{\partial t} \int_{V} \rho \emptyset dV + \oint_{A} \rho \emptyset v. \, dA = \oint_{A} \Gamma_{\emptyset} \nabla_{\emptyset} \cdot dA + \int_{V} S_{\emptyset} dV \tag{2}$$

The structure of this equation is the basis of all CFD methods and shows a transient term followed by a convective, a diffusive and finally the source term. On this equation the continuity, momentum and energy equations can be expressed after the \emptyset variable is respectively changed by the mass, velocity and temperature. For a discrete domain, each cell volume will need to have all of these expressions solved.

OPTIMIZATION

The optimization process enters into the traditional design flow by suggesting that, after proof of performance of the study equipment is realized, modifications are proposed and studied in order to further improve the operation of this project.

The concept stage aims to raise market needs, technological possibilities for implementation in the project and economic feasibility of the investment. After this study, planning is performed for the purpose of mapping the user to the product being developed, their technical requirements, required research, their use and operation as well as specification. Finally, in the stage of product engineering, issues such as design, design calculations, simulations and optimizations, validations and prototyping are discussed to allow this product to be created [3]. After this step, Design begins, at which point the objective esthetics is defined. Then the necessary items are defined, sized and specified. The analysis stage begins, in which simulations will be performed to estimate the product behavior and improvements are proposed if it is found that the operation with the current project is compromised. After this study optimization can be contemplated, process in which objectives and goals are established and modifications are realized in the model with the aim of improving its performance. As this process leads to modifications, new analyzes must be performed and only at the end of this cycle should the product be referred to the generation of a prototype, for validation and finally production, saving unnecessary coasts with intermediate prototypes [4].





Figure 2: Product life cycle development [5].

This cycle can lead to an uncountable number of modifications, which consequently require a countless number of simulations. To avoid infinite possibilities, cases are elaborated using optimization methods. In these methods the variables to be worked are defined, that is to say, which parameters are susceptible of modifications in the model, along with the objectives of this process. At the end of these definitions one must choose an optimization method, which is nothing more than a search technique in which a finite number of cases will be studied and a curve or response surface is generated in order to estimate a possible optimal point of design, or even a point of improvement. After applying these techniques, the analysis of the points obtained for validation of the estimated data with the data obtained by simulation should be performed. This process allows the response to be obtained with a reduced number of simulations and consequently, less time.

Most engineering real problems cannot be expressed in single goal optimization. For multiple-objective cases it is usual that the objectives are conflicting, in order that it is not possible to determine one single optimized configuration [6]. To solve this kind of problem genetic algorithms were developed around 1970 [7]. Genetic Algorithms start their search from a randomly created initial population. From this set of individuals, the adaptation or individual quality of each is analyzed and only a few individuals are preserved for the next generation. The decision criterion is the non-dominated or Pareto-optimal solutions, that is, solutions that cannot be improved independently of the objective unless the worsening of another objective occurs [8]. With the solutions of the population called the parents, crossing and mutation operators are applied in order to generate a new population called a descendant that will be closer to the final response than the previous generation. The process continues in this cycle until the stop condition is reached, as shown in the flowchart elaborated by Deb et al:





Figure 3: Flowchart of Genetic Algorithms [8]

The NSGA method, proposed by Professor Deb and his colleagues [9] is a method of regular accuracy and low convergence rate [10]. However, it is fast and robust, being able to operate continuous and discrete variables and creating new populations in regions statistically more interesting for the calculation [11]. Another advantage of this algorithm is its ability to allow different scenarios to be studied in parallel. For computers with adequate configuration this feature can be a differential to get the problem response in reduced time.

This algorithm was developed to overcome the main criticisms of the models that existed until then, including the NSGA model, which presents high computational cost, the need for an operator called a sharing parameter that raises the level of difficulty of using the technology and the lack of elitist reproduction, which shows itself as a great performance accelerator of genetic algorithms [8].

The main operation starts with a population P_0 . For each solution obtained, a ranking is assigned in ascending order, in which level 1 is the best, followed by level 2 and so on. After the mutation and crossing operations, the population Q_0 is obtained. From the combined population of P_0 and Q_0 , solutions F are obtained from the distances D of each solution for the objectives. Those individuals who present the longest distance are rejected while those of the shortest distance are preserved for the next generation. This process is given by the equation:

$$D = F_{i+1} - F_{i-1}$$

(3)





Figure 4: NSGA-II algorithm proceedings [8]

The elitism concept used in the NSGA-II algorithm is responsible for the disposal of less able individuals on behalf of the population that shows itself more adapted to survive until the next generation, in which it will be combined with other elements giving rise to a new elitist process. Through these operations it becomes possible to select the individuals most suitable for the objectives contemplated.

Finally, in addition to all the qualities presented above, this model was chosen for the current research because of its great expression in the most recent works of optimization of turbo machinery, showing the current trend of the application of optimization algorithms in this specific branch of knowledge. As an example, the work of Zhu et al [12], of Patrao et al [13], of Torabi et al [14], of Zhu et al [15], Wang et al [16] and Wang et al [17] and Amaral et al [18].

COMPUTATIONAL PROCEDURE

The present work makes use of the project developed in previous work [19], in which preliminary data of an axial turbine with generated power of 1,722 MW, adiabatic efficiency of 88.5% and the torque in the axis of 584.092 Nm for the specified gas turbine. This project started at this stage. From the data provided as boundary conditions and the virtual prototype of the turbine, the model was elaborated in the computational solutions using other software for the determination of turbulent 3D flow and optimization.

The steps of modifying the geometry, mesh generation, calculation and analysis of the results were performed at ANSYS tools, while the optimization tool used was Esteco's ModeFRONTIER. The entire study model was made in ANSYS Workbench, which made it possible to generate the model setup in an automated way. Each distinct project that was proposed by ModeFRONTIER was imported by ANSYS, which in turn was able to reproduce all discretization definitions, physical problem definitions and post-processing capabilities for each situation. This robustness is of great importance because it allows a large number of different geometries to be studied in an automated way without tools conflicts of communication and without the necessity of manual scripts created by the user.





Figure 5: CAD model of the rotor and stator of the machine stage [18]

THE USE OF COMPUTATIONAL TOOLS

Initially the CAD model was edited in ANSYS SpaceClaim v18.0 software, an advanced CAD modeling tool to prepare the model for a simulation analysis in which geometric elements unnecessary for the study were eliminated. Then the design was created on Blade Modeler software, a module capable of extracting mathematically the topology of curvature of high complexity of the faces of the profile.

The generation of the fluid domain is created in this step, starting from the geometry of the blades that were properly worked and recreated with the use of parameters. The domain created here has: An inlet; an outlet; an interface region between the stator and rotor domains; walls representing the blades, the hub and the shroud and Faces of periodicity, because the created mathematical domain considers only a stator blade and a blade of the rotor.



Figure 6: CAD model of the rotor and stator of the machine stage [18]

The parameterizable model extracted by Blade Modeler software has another advantage: it can be easily coupled to the Turbogrid software, within the ANSYS



Workbench v.18.0 platform, a tool that has advanced mesh generation capabilities for turbo machinery and allows the generation of hexahedral mesh and of good quality with due refinement, especially in the regions of the leading and trailing edges of both blades. With this, the discrete mathematical model is created.

The mesh generated, shown in Figure 7 is then exported to ANSYS CFX v.18.0 software, a commercial tool in the academic and industrial sector for three-dimensional calculation of computational fluid dynamics (CFD) of turbo machinery and various other engineering problems. The domain of study has an input, an output, two periodic interfaces and an interface to realize the communication of the stator domain with the rotor domain of the rotor. Further details on the mesh will be presented in the Item Mesh Independence.



Figure 7: discrete model of the study domain.

MESH INDEPENDENCE

When modeling a given problem, it is necessary to ensure that the model represents the real case reliably. In this sense, an important analysis is the resolution of the mesh used, since the numerical methods are strongly dependent on the quality of the computational mesh. It is easily observed that the result of a simulation is remarkably sensitive to the number of elements in use. In general, the greater the number of mesh elements that constitute the domain, the greater the level of accuracy of the results obtained. However, overly refined meshes require a lot of computational power for calculation and do not necessarily provide more accuracy. Thus, a strategy is needed that analyzes the independence of the results in relation to the mesh used, so that the mesh less refined and that is able to capture the phenomena of interest is used in the numerical studies.



In this way, it becomes necessary to study mesh convergence (and independence). A study was carried out comparing the results of a physical model variable for different mesh sizes so that it is easy to visualize the point at which we have the ideal mesh, refined enough to capture all the desired phenomena involved, but not overly refined that would delay the study. In this case the value of the Mach number in a region of the trailing edge of the blades and the power generated by the machine is measured. The choice of these data was done because the power is a relevant data and of interest for the study as a whole, while the monitoring of a point variable like Mach is the ideal to better map fluctuations that occur in the domain, since parameter-weighted monitoring such as areas, flow or volumes tend to amortize this characteristic.

Number of elements	Mach	ε%	Power [W]	ε%
45,658	1.154	6.63	1.67588e+06	5.09
50,466	1.186	4.04	1.75423e+06	0.66
2,292,168	1.222	1.13	1.76489e+06	0.06
7,716,375	1.236	0.00	1.76588e+06	0.00
10,216,473	1.236	0.00	1.76588e+06	0.00

Table 1: Mesh	convergence stud	y
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In the possession of all the above data the engineering decision was made to use the 7.7 million element mesh as reference for the presented study, since it presents good reliability with the results and presents a shorter calculation time when compared with the most refined meshes.



RESULTS FROM THE ORIGINAL AXIAL TURBINE

This study considered the response as converged from the point of view of the residuals when the results reached the level of RMS < 10E-6, which is below the standard residual values that are most commonly used in this CFD software, RMS <10E- 4. Also, physics stabilization and numerical balance were also evaluated to ensure convergence. The Pressure contour calculated can be observed at Figure 9. The pressure distribution demonstrates the evolution of this property in the direction of the flow and allows the visualization of the high pressure zone at the entrance of the domain and the reduction of its magnitude in the downstream (stator - rotor) direction. Furthermore, it is possible to view low pressure zones after the trailing edge of the rotor blade.



Figure 9: Pressure contour at the flow path.

Performance data comparing the CFD study with the previous theoretical values is presented through Table 2.

Performance	Design	CFD Original	 	ε (%)
Power [MW]	1.722	1.766	0.044	2.544
Efficiency (%)	88.5	89.8	1.347	1.522
Total pressure ratio	2.160	2.153	0.007	0.338
Torque [Nm]	584.092	599.038	14.946	2.559

Table 2: Comparison of the performance data of the preliminary design with the model used in the CFD study.



RESULTS

For the optimization process, different variations were considered for the blade angles, which were properly parameterized to be used by the genetic algorithm in the evolutionary process. These angles were considered at different heights of each blade. Five heights were used: the base, the top and three more intermediate and equidistant heights of the blade.

For each of the heights described above, seven values for β were used to model the angles of each blade, traversing its leading edge geometry to the trailing edge. From five distinct heights, and seven parameters defined by height there are 35 input parameters for the study of a spade. Since the stage consists of a stator blade and a impeller blade, there are 70 input parameters to be evaluated.

A multiobjective optimization study, a problem with the level of complexity as described above, and the mesh quality already presented considering seventy different parameters, is not feasible to be solved. Because of this reality, different techniques were considered to simplify the proposed problem and thus to obtain data in a shorter time. It was considered the possibility of working with curvature adjustment or response surface techniques, as presented at [20], in which a correlation between the parameters based on the original problem to be studied was created. These strategies were rejected to increase the reliability of the study here, reducing errors with oversimplifications of the mathematical model. However, due to the difficulty of carrying out such a complex study with so many input parameters, a solution already used in other studies that have encountered this same limitation has been adopted, as published by [21]. Sensitivity analyzes were performed to study which angles are most relevant to the expected objectives - to increase power and efficiency, and to allow the optimization process to disregard input parameters.

Before the optimization process, a sensitivity study was proposed to help in understanding the relevance of all the parameters involved and to enable the most interesting ones to be chosen for the defined objectives. For the elaboration of the sensitivity analysis one must create variations of the original design and solve each one of these different simulations. With this, the optimization tool is able to correlate the changes occurring in each new project and to work with election criteria to determine which ones are most pertinent to be worked on. Using the native tool of generating projects with random variations of the original model, respecting a range of possibilities previously defined by the user, more than 1,400 different simulations were performed.

This methodology and procedure allowed the understanding that the base and top angles of the stator and rotor had very low impact on the determined targets, when compared with the other parameters created at intermediate heights of both blades. Due to the low effect of these variables, the decision was made, based on the good practices of turbo machinery projects, to discard them from the optimization process.





The objective of the optimization study is to maximize power and efficiency in turbine operating conditions. The initial population given was achieved by random initialization, taking advantage of individuals created for the sensitivity study, so that a large spectrum could be studied. By this approach, the number of parameters could be reduced from 70 to 49. Then all the projects used for this study were used to feed the heuristic model and serve as the starting point for the optimization process using the NSGA model -II.

The genetic code applied its mechanisms of evolution (crossing and mutation) until the criteria of numerical convergence were reached. Note that the decision to stop in the numerical solution is function of the user experience about the engineering problem being analyzed. Several individuals were determined over the evolutionary generations, which were necessary for the code to reach its convergence. Figure 11 shows all the individuals that were simulated throughout the process. Its color pattern is in function of the number of generations, being dark blue the results of the first generations and red the projects obtained in the last generation of the process.



Figure 11: Individuals calculated by the genetic algorithm.



The Pareto Front obtained, expresses the several possible projects of this machine so that the proposed objectives are reached. The Pareto Frontier demonstrated by Figure 12 shows all the different projects that were obtained from modifications in the original machine, to the detriment of improving each one of the objectives of study. In the vertical axis is expressed the efficiency of the set while in the horizontal axis is the power generated. As expected, there is no single project that meets the proposed objectives and the Pareto Frontier has the function of demonstrating possible modifications that are equivalent to each other, since each project presents greater improvement for one objective to the detriment of the other.



In order to compare different results of this Pareto Border, three different projects were chosen to analyze the results. These projects were named as: Optimized 1: the Pareto Border higher efficiency point; Optimized 2: the "global" point in which a project presents benefits for both efficiency and power generated, in comparison to the original design; Optimized 3: the highest Pareto border power point.

These configurations are used in a refined mesh model to validate the given results for the coarse mesh. As expected, errors can be found from the comparison from models runs with coarse mesh vs the refined one, obtained from the mesh convergence study. As previously explained, due to the time and computational coasts of running the optimization process with the refined mesh would make the study unfeasible. However, the amount of error is within the expected from the previous correlations and the maximum error was in the order of 1%.

Table 3 presents in the next page the comparison between the original turbine design and the three optimized designs, which were selected for post-processing.



Variable	Solution 01		Solution 02		Solution 03	
	Coarse	Refined	Coarse	Refined	Coarse	Refined
Power [W]	1699500	1720970	1805280	1828740	1875450	1892880
Power ε (%)	1,25		1,28		0,92	
Total to static Efficiency [%]	22,925	22,9282	22,416	22,4591	21,8448	21,9543
Efficiency ε (%)	0,01		0,19		0,50	

Table 3: Comparison of the different projects presented.

The design Solution 2 is in the range of possibilities of the Pareto Frontier that surpass the performance of the original project for both objectives. Among the possibilities selected here, this project was more interesting because it privileged both objectives and was selected to be compared in other post processing with the original design.

The blade-to-blade view is an interesting resource for flow analysis along the domain of turbocharged engines. In an analogous way to that used to present the partial results, we opted for the analysis of the results in the solution 2. When analyzing the pressure gradients in a comparative of the original model in relation to the model obtained through the genetic algorithm it becomes clear the greater smoothing and homogenization of the disturbance. The reduction of the shocks found in the rotor blades is also clear, through Figure 13.



Figure 13: Comparison of the static pressure gradient in the original model vs optimized for the blade height of 50%.



CONCLUSIONS

The results obtained from this methodology provided the obtainment of optimized projects in relation to the original machine and verified that the use of genetic algorithm is effective to improve turbo machinery designs. The methodology presented here was able to couple CFD and multiobjective optimization techniques with genetic algorithm effectively and provided adequate results for the improvement of the original machine. With the Pareto frontier obtained, it is up to the designer to define which distinct design will be used based on the various possibilities obtained, in which one output parameter is the most benefited when compared to the other. In this way, the choice will be dependent on the engineering decision to work with a more efficient or more powerful machine.

Multiobjective optimization studies are increasingly present in several sectors of the industry and are present in the most recent researches of fluid dynamics improvement of turbo machinery. This technology makes it possible to obtain relevant data in a timely manner. The process of developing a new axial turbine takes time and optimization algorithms proposes to increase productivity and even to obtain non-intuitive designs in attractive weather, especially in the highly competitive scenario of this industry.

As a suggestion for future studies, the natural evolution for this study would be a new optimization process from the consideration of all input parameters. The sensitivity study proposed here aimed at reducing the total number of parameters that would be used in the analysis to accelerate the results, so that new studies could contemplate all these parameters to verify if this decision would lead to modifications in the Pareto Frontier and until even if there is the possibility of a new Pareto Frontier technically superior to the one found here. Also, fluid structure interaction simulations could also be performed to study heat exchange, flutter and other pertinent phenomena.

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